Elementary Differential and Integral Calculus

FORMULA SHEET

Exponents

$x^a \cdot x^b = x^{a+b}, \quad a^x \cdot b^x = (ab)^x, \quad (x^a)^b = x^{ab}, \quad x^0 = 1.$

Logarithms

$\ln xy = \ln x + \ln y, \quad \ln x^a = a \ln x, \quad \ln 1 = 0, \quad e^{\ln x} = x, \quad \ln e^y = y,$
$a^x = e^{x \ln a}.$

Trigonometry

$\cos 0 = \sin \frac{\pi}{2} = 1, \qquad \sin 0 = \cos \frac{\pi}{2} = 0,$
$\cos^2 \theta + \sin^2 \theta = 1, \qquad \cos(-\theta) = \cos\theta, \qquad \sin(-\theta) = -\sin\theta,$
$\cos(A + B) = \cos A \cos B - \sin A \sin B, \qquad \cos 2\theta = \cos^2 \theta - \sin^2 \theta,$
$\sin(A + B) = \sin A \cos B + \cos A \sin B, \qquad \sin 2\theta = 2 \sin\theta \cos\theta,$
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}, \qquad \sec\theta = \dfrac{1}{\cos\theta}, \qquad 1 + \tan^2 \theta = \sec^2 \theta.$

Inverse Functions

$y = \sin^{-1} x$ means $x = \sin y$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.
$y = \cos^{-1} x$ means $x = \cos y$ and $0 \le y \le \pi$.
$y = \tan^{-1} x$ means $x = \tan y$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.
$y = x^{1/n}$ means $x = y^n$. $\qquad y = \ln x$ means $x = e^y$.

Alternative Notation

$\arcsin x = \sin^{-1} x, \quad \arccos x = \cos^{-1} x, \quad \arctan x = \tan^{-1} x, \quad \log x = \log_e x = \ln x.$
Note: $\quad \sin^{-1} x \neq (\sin x)^{-1}, \quad \cos^{-1} x \neq (\cos x)^{-1}, \quad \tan^{-1} x \neq (\tan x)^{-1}.$
However: $\sin^2 x = (\sin x)^2, \quad \cos^2 x = (\cos x)^2, \quad \tan^2 x = (\tan x)^2.$

Lines

The line $y = mx + c$ has slope m.
The line through (x_1, y_1) with slope m has equation $y - y_1 = m(x - x_1)$.
The line through (x_1, y_1) and (x_2, y_2) has slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and equation $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$.
The line $y = mx + c$ is perpendicular to the line $y = m'x + c'$ if $mm' = -1$.

Circles

The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.
The circle with centre (a, b) and radius r is given by $(x - a)^2 + (y - b)^2 = r^2$.

Triangles

In a triangle ABC:

(Sine Rule) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$; $\qquad$ (Cosine Rule) $a^2 = b^2 + c^2 - 2bc \cos A$.

Pascal's Triangle

$(x+y)^2 = x^2 + 2xy + y^2, \quad (x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ and so on.

The coefficients in $(x+y)^n$ form the nth row of Pascal's triangle:

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
\cdots\cdots\cdots\cdots \\
\text{and so on.}
\end{array}
$$

Quadratics

If $ax^2 + bx + c = 0$, with $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Calculus

If $y = u + v$ then $\dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$. If $y = uv$ then $\dfrac{dy}{dx} = \dfrac{du}{dx}v + u\dfrac{dv}{dx}$.

If $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \left\{\dfrac{du}{dx}v - u\dfrac{dv}{dx}\right\} \Big/ v^2$.

$\displaystyle\int (u+v)\,dx = \int u\,dx + \int v\,dx$. $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int \frac{du}{dx}v\,dx$.

If y is a function of u where u is a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \quad \text{and} \quad \int y\frac{du}{dx}\,dx = \int y\,du.$$

Standard Derivatives and Integrals

If $y = x^a$ then $\dfrac{dy}{dx} = a\,x^{a-1}$, and $\displaystyle\int x^a\,dx = \frac{x^{a+1}}{a+1} +$ constant $\quad (a \neq -1)$.

If $y = \sin x$ then $\dfrac{dy}{dx} = \cos x$, and $\displaystyle\int \sin x\,dx = -\cos x +$ constant.

If $y = \cos x$ then $\dfrac{dy}{dx} = -\sin x$, and $\displaystyle\int \cos x\,dx = \sin x +$ constant.

If $y = \tan x$ then $\dfrac{dy}{dx} = \sec^2 x$, and $\displaystyle\int \tan x\,dx = \ln|\sec x| +$ constant.

If $y = e^x$ then $\dfrac{dy}{dx} = e^x$, and $\displaystyle\int e^x\,dx = e^x +$ constant.

If $y = \ln x$ then $\dfrac{dy}{dx} = \dfrac{1}{x}$, and $\displaystyle\int \frac{1}{x}\,dx = \ln|x| +$ constant.

If $y = \sin^{-1} x$ then $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$, and $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x +$ constant.

If $y = \cos^{-1} x$ then $\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{1-x^2}}$.

If $y = \tan^{-1} x$ then $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$, and $\displaystyle\int \frac{1}{1+x^2}\,dx = \tan^{-1} x +$ constant.